Electroweak Sudakov form factors and nonfactorizable soft QED effects at NLC energies

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Abstract

We study the leading log infrared behavior of electroweak corrections at TeV scale energies, that will be reached by next generation of linear colliders (NLC). We show that, contrary to what happens at typical LEP energies, it is not anymore possible to disentangle "pure electroweak" from "photonic" corrections. This means that soft QED effects do not factorize and therefore cannot be treated in the usual "naive" way they were accounted for in the LEP-era. The nonfactorizable effects come up first at the two loop LL level, that we calculate explicitly for a fermion source that is neutral under the SU(2)⊗U(1) gauge group (explicitly, a Z' decay into two fermions). The basic formalism we set up can be used to calculate LL effects at any order of perturbation theory. The results of this paper might be important for future calculations of electroweak corrections at NLC energies.

1 Introduction

With next generation of linear colliders (NLC [1]), a new era in the testing of electroweak interactions at the quantum level will begin. In fact, when the c.m. energy is much higher than the electroweak scale of about 100 GeV, the pattern of radiative corrections is rather different from what happens at LEP. It turns out that the high energy asymptotic behavior is dictated by the infrared (IR) structure of the theory [2], so that the leading terms grow with the c.m. energy $\sqrt{s}$ like $g^2 \log^2 \frac{\sqrt{s}}{M}$, where g is the gauge coupling and $M_W \approx M_Z \equiv M$ is the weak scale. In refs. [2] and [3] the one loop leading $\propto g^2 \log^2 \frac{\sqrt{s}}{M}$ and subleading $\propto g^2 \log \frac{\sqrt{s}}{M}$ corrections were calculated, while in [4] an expression for the leading terms at all orders in perturbation theory ($\propto (g^2 \log^2 \frac{\sqrt{s}}{M})^n$) has been found. A common feature of the cited works is that they all consider only "pure e.w." corrections to two fermion production, meaning that the photon contribution is not taken into account. However, it is not clear what is the interplay between photon and W,Z bosons contributions at such high energies. For LEP 1-2 [5], the leading infrared photonic (QED) corrections are taken into account separately and factorize with respect to the "hard" corrections, including the mentioned "pure e.w." corrections. One is tempted to extrapolate this approach to higher energies, using what we call a "naive" approach, which would give the following for leading logs and in the limit of massless fermions:

- virtual leading IR QED corrections factorize and exponentiate giving a factor depending on the "photon mass" λ, that acts as a IR cutoff.
- soft photon emission also factorizes, with a factor depending on a typical experimental resolution ΔE.
- "pure ew" virtual corrections can be taken into account separately; at the LL level their effects do not exponentiate trivially but still factorize with respect to the Born level [4].

In this paper we show that this "naive" approach that is correct for LEP energies, is no longer correct when the energies are much higher than the e.w. scale. In the latter case, it is not anymore possible to separate "pure e.w." corrections from photonic corrections. A correct treatment of radiative electroweak corrections is the following:

- *Complete* electroweak virtual corrections are calculated, taking into account also the photon contribution. The photon is given a mass λ to regularize IR divergences.
- soft photon emission is calculated, with photons having energy less than the experimental resolution ΔE

In the limit of massless fermions we consider here, the effect of soft photon emission is basically that of substituting the photon mass λ with the experimental resolution ΔE. Therefore we regularize the IR divergences of the virtual e.w. corrections by giving the photon a mass λ, which has the physical meaning of an experimental resolution on both energies and angles (of order $\frac{\lambda}{\sqrt{s}}$ for the latter). We are thinking about experimental resolutions of the order of $\lambda \approx 10$ GeV, much lower than the W and Z bosons mass so that a process with W- or Z-bremsstrahlung is experimentally resolved. Of course, a realistic calculation taking into account all mass scales and experimental cutoffs, would be much more complicated and goes beyond the scope of this work.

The behavior of electroweak radiative corrections at energies much higher than the electroweak scale is very interesting from a theoretical point of view, since the IR structure of a (spontaneously) broken gauge theory with mixing between the gauge groups has never been considered in the literature. Moreover, from a phenomenological point of view, the planned new generation of linear colliders with TeV scale c.m. energy and very high luminosity [1] should be able to test experimentally such structure.

Bearing in mind simplicity, we consider the LL electroweak corrections to two fermion production by a vector boson behaving like a singlet with respect to the SU(2)$\otimes$U(1) group. To fix ideas, and in order to take a case with phenomenological interest, we study the two fermions decay rate of a massive (mass >1 TeV) Z' gauge boson unmixed with the usual Z boson and belonging to a group which commutes with the SM group. We therefore consider this case as the simplest probe of the infrared structure of electroweak corrections, having in addition phenomenological interest. Moreover, the basic formalism we set up and the general considerations we make are relevant for a more general class of processes of interest at NLC energies.

2 Leading IR electroweak form factor

The tree level amplitude for Z' decay into a $f - \bar{f}$ couple of massless fermions is given by:

$$M_0 \equiv M_0^L + M_0^R = g_L^{Z'}\,\bar{u}(p_f)\gamma^\mu P_L v(p_{\bar{f}})\varepsilon_\mu^*(q) + g_R^{Z'}\,\bar{u}(p_f)\gamma^\mu P_R v(p_{\bar{f}})\varepsilon_\mu^*(q) \tag{1}$$

where $\varepsilon_\mu(q)$ is the polarization of the Z' with momentum $q = p_f + p_{\bar{f}}$ and p_f $(p_{\bar{f}})$ is the fermion (antifermion) momentum. We identify $M_{Z'}^2 \equiv s = 2(p_f \cdot p_{\bar{f}})$ in the following. Since a difference in the masses of Z and W bosons is negligible in LL approximation, we set $M_Z \approx M_W = M = 90$ GeV. In the limit of massless fermions chirality is conserved, so radiative corrections do not mix left and right fermions, that we can consider separately.

In order to compute the leading radiative corrections in the infrared region $\sqrt{s} \gg w \gg M$, where w is the virtual boson energy and M its mass, we use the method of soft insertions formulae, which are widely used in QED [6] and are known to provide in QCD [7, 8] the leading IR singularities at double log level. This method consists in factorizing the softest virtual momentum k^μ by computing external line insertions only, and in iterating this procedure by setting $k = 0$ in the left-over diagram.

Since Goldstone bosons (and ghosts) do not couple to the external massless fermions, we are led to consider only nearly on-shell gauge bosons which are emitted and reabsorbed by an external fermion leg. A gauge boson attaches to a fermion line with an amplitude proportional to the eikonal current $\frac{p_f^\mu}{kp_f}$. We work in Feynman gauge with massless fermions $p_f^2 = p_{\bar{f}}^2 = 0$, so that diagrams in which a boson is emitted and reabsorbed by the same (fermion or antifermion) line do not contribute in leading log approximation.

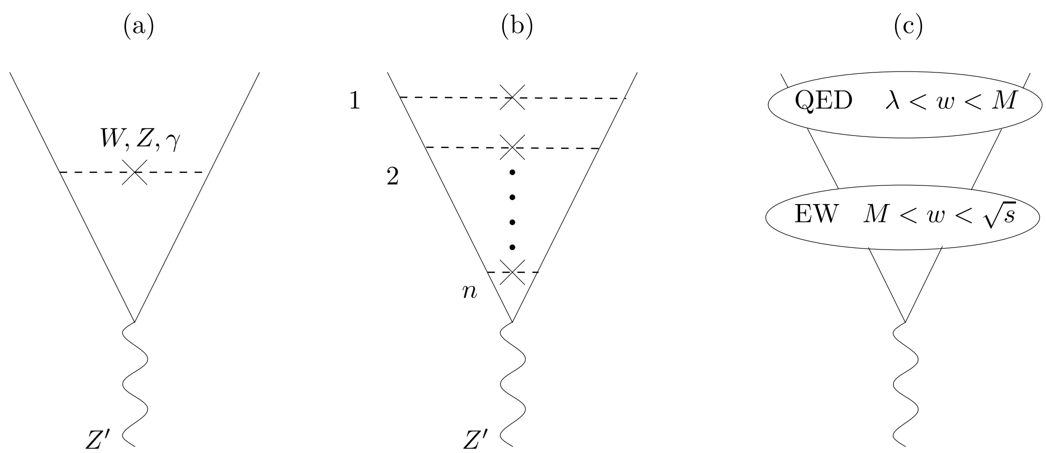

Figure 1: (a)-(b) :diagrams for soft boson insertion at 1 and n loops. Continuous lines are fermion lines, and dashed lines are W, Z, γ gauge bosons.Crosses indicate that gauge bosons are close to mass-shell, and energies are such that $w_1 \ll w_2 \ll ..... \ll w_n$ (see text). (c): Pictorial representation of eq. (11)

Following the calculations explicitly done in the Appendix, we can now write the 1 loop correction to the tree level amplitude (see fig. 1a):

$$\mathbf{M}_1 = -\sum_a \int d[k]\delta(k^2 - M_a^2)\langle f|J_\mu^a(k, p_f)J^{\mu a}(k, p_{\bar{f}})|f\rangle M_0 \qquad J_\mu^a(k, p) = g^a \frac{p^\mu}{(kp)} T^a \tag{2}$$

where $|f\rangle$ is a fermion belonging to a given representation of the SU(2)⊗U(1) gauge group and where we sum over all gauge bosons a. The latter couple to fermions with SU(2)⊗U(1) generators T^a normalized to to $\mathrm{Tr}\{T^aT^b\} = \frac{1}{2}\delta^{ab}$. The charge operator is defined as $Q = T^3 + Y$. We have defined $d[k] = \pi\frac{d^4k}{(2\pi)^4}$; see Appendix

for computational details. Notice that in general, the tree level amplitude M_0 can have a complicated flavor structure *. On the other hand, since the Z' is a completely neutral singlet under SU(2)⊗U(1), the flavor structure is factorized with respect to the tree level amplitude, as shown in (2). This is due to the fact that in this case M_0 is flavor diagonal and is therefore just a number with respect to flavor. Notice also that (2) is just a shorthand notation; in fact the mass eigenstates (Z, γ) do not coincide with the gauge bosons vectors A_3, B. Thus we have to take into account this mixing in the neutral sector (see later).

At higher orders, we assume that the soft boson insertion formula can be iterated at the LL level in the "strong energy ordering region" [8]. This means that the soft bosons energies labelled from 1 (external boson) to n (innermost boson) are such that $w_1 \ll w_2 \ll ..... \ll w_n$ (see fig. 1b). Then, the two loop contribution is given simply by:

$$\mathbf{M}_2 = \sum_{a,b} \int d[k_1]d[k_2]\delta(k_1^2 - M_a^2)\delta(k_2^2 - M_b^2)\langle f|J^a_\mu(k_1, p_f)J^b_\nu(k_2, p_f)J^{\nu b}(k_2, p_{\bar f})J^{\mu a}(k_1, p_{\bar f})|f\rangle M_0 \tag{3}$$

and so on for the higher order contributions. In order to write the n-th order contribution in a compact form we make a slight change in the notation and we define the operators $J^{\mu a}(k) = g^a \frac{p_f^\mu T^a}{kp_f}$, $\tilde{J}^{\mu a}(k) = -g^a \frac{p_{\bar f}^\mu \tilde{T}^a}{kp_{\bar f}}$. The "untilded" operators act on the left as usual, i.e. $J^{\mu a}(k)\ M = g^a \frac{p_f^\mu}{kp_f}\ T^a M$, while the "tilded" operators act on the right, i.e. $\tilde{J}^{\mu a}(k)\ M = g^a \frac{p_{\bar f}^\mu}{kp_{\bar f}}\ MT^a$. Equation (3) can therefore be rewritten as:

$$\mathbf{M}_2 = \sum_{a,b} \int d[k_1]d[k_2]\delta(k_1^2 - M_a^2)\delta(k_2^2 - M_b^2)\langle f|J^a_\mu(k_1)\tilde{J}^{\mu a}(k_1)J^b_\nu(k_2)\tilde{J}^{\nu b}(k_2)|f\rangle M_0 \tag{4}$$

In eikonal approximation, before integrating over $k_1, k_2.....k_n$, the n-th order matrix element is obtained from the matrix element of order n-1 by insertion of the following operator $I(k)$:

$$M_n = (-)^n \sum_a \delta(k^2 - M_a^2)J^{\mu a}(k)M_{n-1}J^{\mu a}(k) \equiv I(k)M_{n-1} \qquad I(k) = \sum_a J^{\mu a}(k)\tilde{J}^{\mu a}(k)\delta(k^2 - M_a^2) \tag{5}$$

Let us now calculate the insertion operator taking into account also gauge bosons mixing in the neutral sector. The insertion of a W boson gives $g^2(T^1\tilde{T}^1 + T^2\tilde{T}^2)$ while inserting a Z boson gives $\frac{g^2}{c_w^2}(T^3 - s_w^2 Q)(\tilde{T}^3 - s_w^2\tilde{Q}) = g^2 T^3\tilde{T}^3 + g'^2 Y\tilde{Y} - e^2 Q\tilde{Q}$. By defining $\bar{T}\cdot\bar{\tilde{T}} = \sum_i T_i\tilde{T}_i$ and since we take the Z and W to be degenerate at the same mass M, we can write:

$$I(k) = \frac{p_f p_{\bar f}}{(p_f k)(p_{\bar f} k)}\left\{[g'^2 Y\tilde{Y} + g^2\bar{T}\cdot\bar{\tilde{T}} - e^2 Q\tilde{Q}]\delta(k^2 - M^2) + e^2 Q\tilde{Q}\delta(k^2 - \lambda^2)\right\} \tag{6}$$

The integrated matrix element is therefore:

$$M_n = \langle f|\int d[k_1]d[k_2]...d[k_n]I(k_1)I(k_2)...I(k_n)|f\rangle M_0 \qquad w_1 \ll w_2 \ll .... \ll w_n \tag{7}$$

For every $k_i = (w_i, \mathbf{k}_i)$ we can now integrate over the collinear regions $\mathbf{k}_i \cdot \hat{\mathbf{p}}_f \sim 1$, $\mathbf{k}_i \cdot \hat{\mathbf{p}}_{\bar f} \sim 1$ at fixed w_i. This gives factors $\log\frac{w_i}{M_a}$ (see Appendix). The subsequent integral is over $\frac{dw_i}{w_i} = d\log w_i$. It is therefore natural to use the logarithm of the energy as a variable, defining:

$$l = \log\frac{M}{\lambda} \quad L = \log\frac{\sqrt{s}}{M} \quad x_i = \log\frac{w_i}{M} \tag{8}$$

The collinear integrals give then $\log\frac{w}{\lambda} = x + l$ for the photon and $\log\frac{w}{M} = x$ for massive bosons. From eqns (6,7) we obtain:

*here and in the following, by "flavor" we mean SU(2)⊗U(1) quantum numbers; we only consider a single generation of fermions

$$\int d[k]I(k) = -\int dx H(x); \quad H(x) = \Theta_0^L[g'^2 Y\tilde{Y} + g^2 \bar{T}\cdot\bar{\tilde{T}} - e^2 Q\tilde{Q}]x + \Theta_{-l}^L e^2 Q\tilde{Q}(x+l) \tag{9}$$

where Θ_a^b is 1 for $a < x < b$, 0 elsewhere. Notice that here we have for sake of simplicity, reabsorbed all the constants coming from phase space and integral over the angles in a redefinition of the couplings; that is, we have made the substitution $\frac{g_a^2}{2\pi^2} \to g_a^2$ (see Appendix). We rewrite eq. (9) in a slightly different form:

$$\begin{aligned} H(x) &= \Theta_{-l}^0 H_{QED}(x) + \Theta_0^L H_{EW}(w) \\ H_{QED}(x) &= e^2 Q\tilde{Q}(x+l) \qquad H_{EW}(x) = (g'^2 Y\tilde{Y} + g^2\bar{T}\cdot\bar{\tilde{T}})x + e^2 Q\tilde{Q}l \end{aligned}$$

Since the leading terms are given by the regions of phase space strongly ordered in energy [8], the overall form factor is given by:

$$\sum_{n=0}^{\infty}\mathbf{M}_n = \sum_{n=0}^{\infty}(-)^n\langle f|\int dx_1 dx_2 \ldots . dx_n \underbrace{H(x_1)H(x_2)\ldots . H(x_n)}_{x_1<x_2<\ldots .<x_n}|f\rangle M_0 \equiv \langle f|P_x\{\exp[-\int dx H(x)]\}|f\rangle M_0 \tag{10}$$

The first thing to notice is that the total form factor can be written

$$P_x\{\exp[-\int_{-l}^{L} dx H(x)]\} = P_x\{\exp[-\int_{-l}^{0} dx H_{QED}(x)]\} \times P_x\{\exp[-\int_0^L dx H_{EW}(x)]\} \tag{11}$$

Since $[H_{QED}(x_1), H_{QED}(x_2)] = 0$, the x-ordered exponential turns into a regular exponential for $-l < x < 0$ and we have:

$$\langle f|P_x\{\exp[\int_{-l}^{0} dx - H_{QED}(x)]\}P_x\{\exp[-\int_0^L dx H_{EW}(x)]\}|f\rangle = \exp[-e^2 q_f^2\frac{l^2}{2}]\langle f|P_x\exp[-\int_0^L dx H_{EW}(x)]|f\rangle \tag{12}$$

We conclude that the total form factor is given by †:

$$\boxed{\sum_{n=0}^{\infty}\mathbf{M}_n = \exp[-e^2 q_f^2\tfrac{l^2}{2}]\sum_{n=0}^{\infty}M_n = \exp[-e^2 q_f^2\tfrac{l^2}{2}]\langle f|P_x\exp[-\int_0^L dx H_{EW}(x)]|f\rangle M_0} \tag{13}$$

$$\boxed{H_{EW}(x) = (g'^2 Y\tilde{Y} + g^2\bar{T}\cdot\bar{\tilde{T}})x + e^2 Q\tilde{Q}l} \tag{14}$$

These formulae set up the basic formalism allowing to calculate the effects we are interested in. Here a separation of scales is implicit, in such a way that

- For energies $\lambda < \sqrt{s} < M$ only the photon propagates, i.e. $H(x) = H_{QED}(x)$. Therefore QED effects still factorize and exponentiate, but this is true only up to an energy M.
- The contribution of photons with energies higher than M is taken into account in a nontrivial way in H_{EW}. The fact that the Z and γ bosons have completely different mass scales M, λ produces an incomplete cancellation of collinear divergences in the logarithmic term $l = \log\frac{w}{\lambda} - \log\frac{w}{M}$ in (14).

This situation is depicted in fig. 1c. All photons with energies between λ and M stand in the external "blob" and form an abelian factorized structure. In the internal blob instead all bosons W, Z, γ with $M < w < \sqrt{s}$ propagate.

Let us now concentrate on that part of the form factor given by H_{EW}. At the 1 loop level we obtain:

$$M_1 = -\langle f|\int_0^L dx[(g'^2 Y^2 + g^2\bar{T}^2)x + e^2 Q^2 l]|f\rangle M_0 = -(a_f\frac{L^2}{2} + b_f lL)M_0 \tag{15}$$

†Operators are in capital letters, c-numbers in small letters. Thus Q is an operator with values $q_e = -1, q_\nu = 0$

where a_f and b_f are the c-numbers[‡]:

$$a_f = \langle f|g^2\bar{T}^2 + g'^2 Y^2|f\rangle \qquad b_f = \langle f|e^2 Q^2|f\rangle \tag{16}$$

At 2 loops the algebra gets a bit more complicated. We have to calculate:

$$M_2 = \langle f|P_x \int_0^L dx H_{EW}(x_1)H_{EW}(x_2)|f\rangle M_0 = \langle f| \int_0^L dx_2 \int_0^{x_2} dx_1 H_{EW}(x_1)H_{EW}(x_2)|f\rangle M_0 \tag{17}$$

The operator with the highest value of x in the x-ordered product, $H_{EW}(x_2)$ in this case, corresponds to the more internal boson in fig. 1. Therefore, for this operator we can substitute $\tilde{T}^a \to T^a \forall a$. This is of course not true for $H_{EW}(x_1)$. We have:

$$\begin{aligned} H_{EW}(x_1)H_{EW}(x_2) &= H_{EW}(x_1)[(g'^2Y^2 + g^2\bar{T}^2)x_2 + e^2Q^2 l] \\ = [(g'^2Y^2 + g^2\bar{T}^2)x_2 + e^2Q^2 l][g'^2Y^2 x_1 + e^2Q^2 l] &+ \sum_{a=1}^{3} g^2 x_1 T^a [(g'^2Y^2 + g^2\bar{T}^2)x_2 + e^2Q^2 l]T^a \\ &= [(g'^2Y^2 + g^2\bar{T}^2)x_2 + e^2Q^2 l][(g'^2Y^2 + g^2\bar{T}^2)x_1 + e^2Q^2 l] - 2e^2g^2 lYx_1T^3 \end{aligned} \tag{18}$$

where we have used the commutator algebra:

$$[((g'^2Y^2 + g^2\bar{T}^2)x_2 + e^2Q^2 l), Q] = [((g'^2Y^2 + g^2\bar{T}^2)x_2 + e^2Q^2 l), Y] = 0 \tag{19}$$

$$[T^a, (g'^2Y^2 + g^2\bar{T}^2)x_2 + e^2Q^2 l] = e^2 l[T^a, Q^2] = -2e^2 lYi\varepsilon^{3ab}T^b \tag{20}$$

Using (18) we easily obtain:

$$M_2 = \left\{ \langle f|P_x \int_0^L dx_1 dx_2 H_{EW}(x_1)H_{EW}(x_2)|f\rangle \right\} M_0 = \left\{ \frac{1}{2}(a_f \frac{L^2}{2} + b_f lL)^2 - \frac{1}{3}e^2g^2 lL^3 y_f t_f^3 \right\} M_0 \tag{21}$$

It should be evident from the above that there is in general no simple all-orders exponentiation of the LL effects. This is due to the combined effect of the nonabelian group structure and of the gauge bosons mixing in the neutral sector. Moreover, what we see is that there is an interesting interplay between the Z and γ contributions. In particular, the term in (14) proportional to $Q\tilde{Q}$ is nonzero due to a non complete cancellation of collinear divergences cutoffed by two different mass scales M and λ.

3 Comparing with the "naive" approach

We now wish to compare our approach with the "naive" approach where QED corrections are taken into account separately. They are supposed to factorize and exponentiate with a factor [6] $\exp[-e^2q_f^2\log^2\frac{\sqrt{s}}{\lambda}] = \exp[-e^2q_f^2(l+L)^2]$. We concentrate therefore on the "pure electroweak" part of the corrections. These include only Z and W contributions, so that:

$$\mathcal{M}_n = \langle f| \int d[k_1]d[k_2]...d[k_n]\mathcal{I}(k_1)\mathcal{I}(k_2)...\mathcal{I}(k_n)|f\rangle \quad \mathcal{I}(k) = \frac{p_f p_{\bar{f}}}{(p_f k)(p_{\bar{f}} k)}[g'^2Y\tilde{Y} + g^2\bar{T}\cdot\bar{\tilde{T}} - e^2Q\tilde{Q}]\delta(k^2 - M^2)$$

$$\int d[k]\mathcal{I}(k) = -\int dx\mathcal{H}(x); \quad \mathcal{H}(x) = [g'^2Y\tilde{Y} + g^2\bar{T}\cdot\bar{\tilde{T}} - e^2Q\tilde{Q}]x \tag{22}$$

Since $[\mathcal{H}(x_1), \mathcal{H}(x_2)] = 0$, $\int_0^L \mathcal{H}(x)dx$ exponentiates as an operator, and the "pure e.w." form factor is given by:

$$\mathcal{M} = \langle f| \exp\left\{ -[g'^2Y\tilde{Y} + g^2\bar{T}\cdot\bar{\tilde{T}} - e^2Q\tilde{Q}]\frac{L^2}{2} \right\} |f\rangle M_0 \tag{23}$$

[‡] for the fundamental representation of fermions, $\bar{T}^2 = \sum_i T_i T_i = \frac{3}{4}$

We would like to stress the fact that the form factor exponentiates *as an operator* and not as a number. Again, the reason of non exponentiation are noncommutativity and mixing. However, (22) misses the presence of the scale λ which is present in (14). By using commutator algebra, we obtain in this case the following results at 1 and 2 loop level:

$$\mathcal{M}_1 = \langle f| - [g'^2 Y^2 + g^2 \bar{T}^2 - e^2 Q^2]\frac{L^2}{2}|f\rangle M_0 = -(a_f - b_f)\frac{L^2}{2} M_0 \tag{24}$$

$$\mathcal{M}_2 = \frac{1}{2}\langle f|[(g'^2 Y\tilde{Y} + g^2 \bar{T}\cdot\bar{\tilde{T}} - e^2 Q\tilde{Q})\frac{L^2}{2}]^2|f\rangle M_0 = \frac{1}{2}(\frac{L^2}{2})^2[(a_f - b_f)^2 + 2e^2 g^2 y_f t_f^3] M_0 \tag{25}$$

where a_f and b_f are defined in (16).

To compare the results of the previous sections, we have to include properly QED effects that factorize in a different way in the two different approaches; thus we define:

$$\Delta \equiv \exp[-e^2 q_f^2 \frac{l^2}{2}](\sum_{i=0}^{\infty} M_n) - \exp[-e^2 q_f^2 \frac{(l+L)^2}{2}](\sum_{i=0}^{\infty} \mathcal{M}_n) = \sum_{i=1}^{\infty}\Delta_i \qquad \mathcal{M}_0 = M_0 \tag{26}$$

with the self-explaining convention that Δ_i is the difference of order $(g^2)^i$ between the "naive" and the "correct" approach. From (15,24,26) we see that $\Delta_1 = 0$ both for left and right fermions. This was to be expected since at the one loop level γ, Z, W contributions can all be treated separately. At higher orders, let us now consider separately left and right final fermions.

In the right fermions case, everything is proportional to the charge operator Q and therefore the operator algebra is commuting; we call this the abelian case. It is easy to show that for right fermions $\Delta = 0$ to all orders. In fact in this case the abelian structure allows for exponentiation of the one loop result, giving:

$$\exp[-e^2 q_f^2 \frac{l^2}{2}]\exp[-(g'^2 q_f^2 \frac{L^2}{2} + e^2 q_f^2 lL)] M_0^R = \exp[-e^2 q_f^2 \frac{(l+L)^2}{2}]\exp[-(g'^2 q_f^2 - e^2 q_f^2)\frac{L^2}{2}] M_0^R \tag{27}$$

i.e., the same result with the two different approaches.

On the other hand, in the case of left handed fermions the second order value of Δ is different from zero:

$$\Delta_2 = -e^2 g^2 y_f t_f^3 (\frac{L^4}{4} + \frac{L^3 l}{3}) \tag{28}$$

Several comments are in order. In first place, we have seen that this expression arises mathematically from the fact that the operators Q and T^1, T^2 do not commute. Physically this corresponds to the fact that the insertions of a W and of a photon do not commute due to the change of flavor. The second observation is that Δ_2 contains a term proportional to L^4. Now, the limit $\frac{l}{L} \to 0$ (or, equivalently, $\sqrt{s} \to \infty$) can be physically interpreted as the unbroken symmetry limit, in which the whole group SU(2)⊗U(1) exponentiates. However, we see from eq (28) that Δ_2 doesn't vanish in this limit; something must be wrong with one of the two approaches. The all orders formula in our approach can be easily evaluated in this limit

$$\exp[-q_f^2 \frac{l^2}{2}]\langle f|P_x \exp\int_0^L dx H_{EW}(x)|f\rangle M_0^L \xrightarrow{l\to 0} \langle f|P_x \exp\int_0^L dx[g'^2 Y\tilde{Y} + g^2 \bar{T}\cdot\bar{\tilde{T}}]x|f\rangle M_0^L \tag{29}$$

and gives the exponential of the SU(2)⊗U(1) group Casimir, $\exp[-(g'^2 y_f^2 + \frac{3}{4}g^2)\log^2\frac{\sqrt{s}}{M}]$. On the contrary, from formula (28), we see that the "naive" approach gets a different result already at the 2 loop level.

We can summarize the situation like this:

- In the abelian case, the form factor is a regular exponential and we get the same results with the two approaches.
- in the non abelian case, beginning from the 2 loop level there is a difference due to commutator algebra.
- The "naive" approach doesn't get the right result in the limit $\sqrt{s} >>> M$ where we expect that SU(2)⊗U(1) factorizes as properly accounted for by our approach.

To conclude, we observe that the numerical value of Δ depends on the value of λ. However, for typical values $\sqrt{s} \sim 1$ TeV, $\lambda \sim 10$ GeV, the difference between the two approaches is $\Delta_2 \approx 2 \times 10^{-3}$ [§] and is therefore non negligible in view of the expected experimental accuracy of a few *permille* at NLCs.

4 Conclusions

We have investigated the leading IR behavior of radiative electroweak corrections at energies much higher than the electroweak scale. The "naive" expectation of a pattern similar to the LEP case, with QED soft effects factorizing in an independent way from the rest of "pure e.w." corrections, turns out to be incomplete. Instead, photonic virtual effects only exponentiate up to a scale M, which is of the order of the W and Z bosons mass. Above M, the effects of mixing in the neutral sector and the presence of two mass scales for the photon on one side and W, Z bosons on the other have to be taken into account properly. We have established a formalism allowing to compute the LL electroweak corrections at any order, and calculated explicitly the second order effect in the case under exam. We have seen two different reasons for the difference with the "naive" approach: one is the noncommutativity of SU(2)⊗U(1) generators and the other is spontaneous symmetry breaking, which induces mixing in the neutral sector and generates two different mass scales for the gauge bosons.

In this paper we have considered the simple case of a process with two fermions on the external legs coupling to a vector boson which is neutral under the SU(2)⊗U(1) gauge group. Cases of more immediate phenomenological interest, like processes with 4 fermions on the external legs (for instance, $e^+e^- \to \mu^+\mu^-$), that have a more complicated flavor structure and a higher number of diagrams, are currently under study. However, the basic formalism we have set up and the general considerations we have made are relevant for a large class of processes of interest at NLC energies.

In general, for processes of relevance for NLCs we expect a difference between our approach and the "naive" one at the 2 loop LL level, of the order $(\frac{g^2}{16\pi^2}\log^2\frac{s}{M^2})^2 \sim$ a few *permille*. Since this difference arises at the leading log level, any future calculation of higher order leading IR electroweak corrections at TeV scale energies will have to cope with spontaneous symmetry breaking, mixing, and with the presence of two gauge bosons scales.

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A Appendix

Let us consider the emission of a boson with momentum $k = (w, \mathbf{k})$, $k^2 = M^2$ from a fermion with momentum $p_f = \frac{\sqrt{s}}{2}(1, \hat{\mathbf{p}}_f)$, $p_f^2 = 0$. The denominator of the fermion propagator after the emission is given by:

$$(p_f + k)^2 = M^2 + w\sqrt{s}\left(1 - \frac{\sqrt{w^2 - M^2}}{w}\cos\theta\right) \approx \frac{w\sqrt{s}}{2}\left(\theta^2 + \frac{M^2}{w^2} + \frac{2M^2}{w\sqrt{s}}\right) \approx \frac{w\sqrt{s}}{2}\left(\theta^2 + \frac{M^2}{w^2}\right) \tag{30}$$

where $\cos\theta = \hat{\mathbf{k}}\hat{\mathbf{p}}_f$ and where we use the fact that the leading logs come from the region $M \ll w \ll \sqrt{s}, \theta \approx 0$. The collinear (lower) cutoff is therefore simply given by $\theta^2 > \frac{M^2}{w^2}$. The integral relevant for the one loop result,

[§] we remind the reader that couplings have been renormalized, so one should substitute $g_a^2 \to \frac{g_a^2}{2\pi^2}$ in formulae like (28) to get the results with the usual definitions of couplings

whose leading (double log) behavior is dictated by the regions of k where the boson is on shell, is given by

$$i\frac{g^2}{(2\pi)^4}\int \frac{d^4k}{(k^2-M^2+i\varepsilon)}\frac{4(p_f\cdot p_{\bar{f}})}{(k^2+2p_f\cdot k)}\frac{1}{(k^2+2p_{\bar{f}}\cdot k)} \approx \frac{g^2}{(2\pi)^4}\pi(p_f\cdot p_{\bar{f}})\int \frac{\delta(k^2-M^2)d^4k}{(p_f\cdot k)(p_{\bar{f}}\cdot k)} \tag{31}$$

$$= \frac{g^2}{(2\pi)^4}\pi(p_f\cdot p_{\bar{f}})\int \frac{d^3\mathbf{k}}{w}\frac{1}{(p_f\cdot k)}\frac{1}{(p_{\bar{f}}\cdot k)}\bigg|_{w=\sqrt{|\mathbf{k}|^2+\mathbf{M}^2}} = \frac{g^2}{(2\pi)^4}\pi^2\int\frac{|\mathbf{k}|^2\mathbf{d}|\mathbf{k}|}{w^3}\int\frac{d\theta^2}{(\theta^2+\frac{M^2}{w^2})} \tag{32}$$

$$= -\frac{g^2}{16\pi^2}\int_M^{\sqrt{s}}\frac{dw}{w}2\log\frac{w}{M} = -\frac{g^2}{16\pi^2}\int_0^{\log\frac{\sqrt{s}}{M}} d(\log^2\frac{w}{M}) = -\frac{g^2}{16\pi^2}\log^2\frac{\sqrt{s}}{M} \tag{33}$$

The integral receives two equal contributions from the regions where $\cos\theta = \hat{\mathbf{k}}\hat{\mathbf{p}}_f \approx 1$ and where $\cos\bar{\theta} = \hat{\mathbf{k}}\hat{\mathbf{p}}_{\bar{f}} \approx 1$ Moreover, since $\delta(k^2-M^2)$ has two poles, we have to consider also $w = -\sqrt{|\mathbf{k}|^2+\mathbf{M}^2}$. This second pole gives the same results, but in different angular regions ($\cos\theta \to -\cos\theta$). In the end we have to multiply by a factor 4. The final result is $g^2/(4\pi^2)\log^2\frac{\sqrt{s}}{M}$. This result agrees with the one obtained in [2] by calculating the C-functions asymptotic behavior. In eq. (9), we redefine $g^2/(2\pi^2) \to g^2$ and this result is written $g^2/2\log^2\frac{\sqrt{s}}{M}$.

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